Verification of the VirtuCath Steering Simulation

The quasi-static steering solver checked against exact solutions, a refinement study and a Cosserat-rod reference model

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Summary

VirtuCath's steering simulation predicts how a catheter bends when a pull wire is drawn back: the shape, the tip angle and the wire tension. It takes the stiffness of each shaft section and finds the catheter's static equilibrium. This report checks that the solver behind it solves its equations correctly. The section stiffnesses it uses are validated separately, in *Validation of the VirtuCath Stiffness Model*.

0.063 %	0.51 %	0.0061 °	949
median error against exact beam theory, small deflection	median departure from the exact arc, large deflection	median tip-angle error against the Cosserat-rod model	catheters compared with the reference model

- **Exact solutions:** for a segment bending under its own weight, the solver matches classical beam theory with a median error of 0.063 % in tip deflection. For a segment bent by a pull wire, whose exact shape is a circular arc at any deflection, it departs from the exact tip angle by a median of 0.51 %.
- **Reference model:** over 949 catheters of one and three segments, bent by up to 180°, the median tip-angle error against a published Cosserat-rod model was 0.0061 ° and 0.0035 °, and the median wire-tension error 0.52 % and 0.47 %.
- **Where the error comes from:** the solver represents the catheter as a chain of short rigid bodies. Its remaining error comes from that discretization, and it falls steadily as the chain is refined.
- **Not yet done:** a comparison of predicted shapes with physical catheters (Section 5).

> **In short.** The steering solver solves its mechanics correctly. For a given pull, its tip angle agrees with a high-fidelity reference to a few hundredths of a degree and its wire tension to about half a per cent. Its remaining error is a small, systematic discretization error that halves each time the chain of bodies is refined twofold.

1 The steering solver

The solver represents the catheter as a chain of rigid bodies joined by bending, torsion and axial springs, with stiffnesses from each section's *EI*, *GJ* and *EA*. The catheter is divided into 100 bodies, placed more densely where it is more flexible and where the pull wires bend it, as in the app's Steering tab. A pull wire runs through a guide at each joint between bodies, at its offset from the centreline, so its moment arm about each joint is that offset (times the cosine of the joint's bend). It carries tension only, never compression.

For a given pull, the solver computes the catheter's static equilibrium.

Intended use. Like the rest of VirtuCath, the steering simulation supports early and mid design: screening and comparing designs. VirtuCath does not replace design verification tests.

2 How the solver was checked

The checks follow ASME V&V 10 [1] and answer five questions (Table 1), from the code up to physical catheters.

Table 1. The evidence at a glance.

Question	How it was checked	Result	Status
1. Does the code solve its equations correctly?	Derivatives against finite differences; parity with the previous engine; release, stepping and refinement tests	All within tolerance	**Pass**
2. Does it reproduce exact solutions?	Cantilever under its own weight (500 cases); constant-curvature arc under a pull wire (474 cases)	Median 0.063 % and 0.51 %	**Pass**
3. Is the numerical error small?	Every tenth case re-solved with a finer chain of bodies	Median changes of 1 % or less	**Pass**
4. Does it agree with a high-fidelity reference model?	Cosserat-rod model [3], 474 one-segment and 475 three-segment catheters	Median tip angle 0.0061 ° and 0.0035 °	**Pass**
5. Does it agree with physical catheters?	Measured tip angle and wire tension on built shafts	–	Future work

3 Results

3.1 The code solves its equations correctly

The solver uses analytic first and second derivatives of the catheter's energy. On a catheter bent to about 100° with the pull wire taut and gravity on, both agree with finite differences of the energy: the gradient to 3×10^{-8} and the Hessian to 3×10^{-9} of their largest entries, and the Hessian is exactly symmetric. Further automated tests (Table 2) check the solver against the previous simulation engine and against basic behaviour.

Table 2. Code verification. All tests pass on VirtuCath 1.7.5.

Check	Result	Criterion
Gradient against central differences (300 unknowns)	3×10^{-8}	10^{-6}
Hessian against differences of the gradient	3×10^{-9}	10^{-6}
Hessian symmetry	exact	10^{-9}
Tip angle against the previous engine (pulls of 0.5 to 4.5 mm)	within 2×10^{-3} °	2×10^{-3} °
Wire tension against the previous engine	within 2×10^{-4} relative	2×10^{-4}
Release of the pull wire returns the catheter straight	holds	–
Gradual drive ends at the direct static solution	holds	–
Refining the chain of bodies converges	holds	–

3.2 It reproduces exact beam theory in small deflection

For a uniform segment of length L and bending stiffness EI, clamped at one end and bending under its own weight w per unit length (Fig. 1), linear beam theory gives the tip deflection and angle exactly [2]:

$$\delta = \frac{wL^4}{8\,EI}, \qquad \theta = \frac{wL^3}{6\,EI}. \tag{1}$$

500 segments were drawn at random, uniformly over length (10 to 100 mm) and stiffness (1.0×10^{-4} to $1.0 \times 10^{-2}\,\mathrm{N\,m^2}$), with the weight chosen to keep the tip deflection under 5 % of the length, where the formulas are exact (largest 5.0 %).

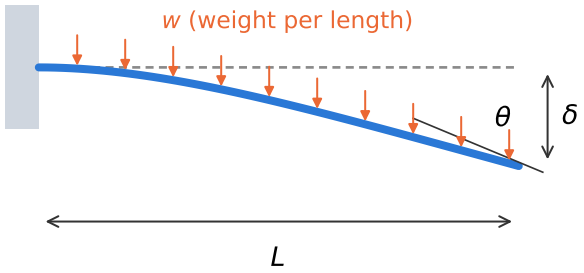

Figure 1. The small-deflection benchmark: a uniform segment clamped at the left, bending under its own weight w per unit length. The tip deflects by δ and rotates by θ.

Result. The solver matches beam theory for all 500 segments (Table 3): a median error of 0.063 % in tip deflection and 0.051 % in tip angle.

Table 3. Error of the solver against beam theory over 500 segments.

	Tip deflection		Tip angle	
Measure	mm	%	°	%
Median absolute error	0.00085	0.063	0.0012	0.051
P95 absolute error	0.0053	0.17	0.0049	0.13
Largest absolute error	0.0086	0.19	0.0055	0.15
Bias (mean signed error)	-0.0015	-0.072	-0.0017	-0.058

The remaining difference is the linear theory's. Each segment has 60 to 100 bodies (median 100). At that resolution the solver is closer to the exact solution of the bending problem than the linear formulas are: they are exact only in the limit of vanishing deflection. The difference grows with the deflection ratio δ/L (Fig. 2): a median of 0.0086 % in deflection below 2 %, 0.052 % from 2 to 3.5 % and 0.13 % from 3.5 to 5 %. In 97 % of segments the solver deflects slightly less than the linear formula, as a large-deflection solution does. Refining the chain does not reduce it: on every tenth segment the median difference in deflection is 0.072 % at 100 bodies, 0.080 % at 200 and 0.081 % at 366. The discretization error is smaller still (Table 4).

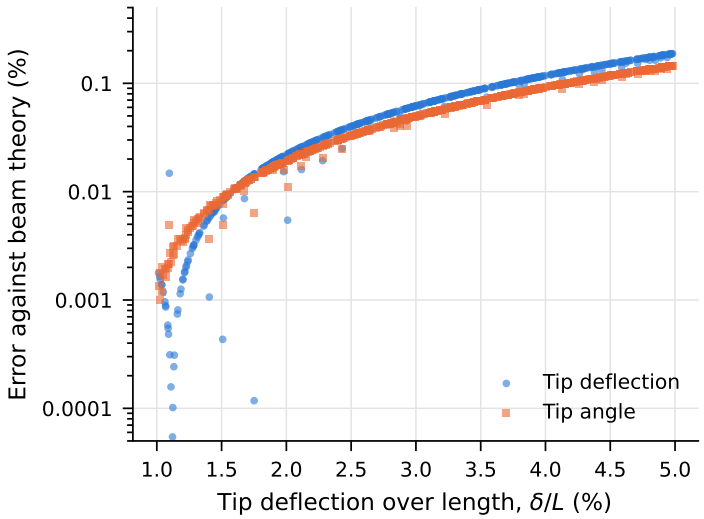

Figure 2. Difference from beam theory against the deflection ratio δ/L, for each segment.

3.3 It reproduces the exact arc in large deflection

A uniform segment bent by a pull wire at a constant offset d carries the same bending moment Fd along its whole length, where F is the wire tension. It therefore bends into a circular arc, and its tip angle is

$$\theta = \frac{F\,d\,L}{EI} \tag{2}$$

at any deflection, not only small ones. This gives an exact check in large deflection, using each model's own wire tension.

Result. Over the 474 one-segment catheters of Section 3.5, the solver's tip angle departs from the exact arc by a median of 0.51 % (P95 0.55 %). The departure is systematic: in 100 % of catheters the solver's segment bends slightly less than the exact arc for the same tension. A chain of N bodies carries the segment's bending compliance at its joints, and the last half body beyond the last joint bends only with the body it belongs to, so the chain is stiffer than the segment by about $1/(2N)$. The departure therefore halves each time the chain is refined twofold (Fig. 3): 0.51 % at the standard budget (median 100 bodies), 0.25 % with about twice as many bodies and 0.13 % with about four times as many. For comparison, the Cosserat-rod reference meets the same relation to 0.00036 %.

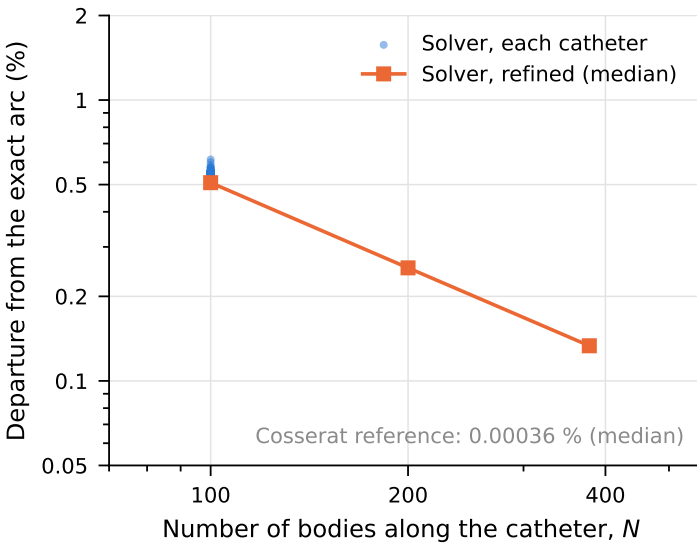

Figure 3. Departure of the solver's tip angle from the exact arc, against the number of bodies. Dots: each one-segment catheter at the standard budget. Squares: median over every tenth catheter re-solved with about two and four times as many bodies. The note gives the Cosserat-rod reference's median for comparison.

3.4 The numerical error is small

Every tenth case of each study was solved again with a finer chain of bodies (Table 4). Refining changes the results by a median of 1 % or less, which bounds the typical discretization error at the standard budget. The largest change, 1.8 % in the wire tension of one three-segment catheter, is the only one above 1 %.

Table 4. Change in the solver's output when the chain of bodies is refined, from the standard budget to the finest level run (median bodies in brackets). Median and largest change over every tenth case.

Study	Output	Median change	Largest change
Small deflection (100 → 366)	Tip deflection	0.0092 %	0.0094 %
	Tip angle	0.0040 %	0.0064 %
One segment (100 → 378)	Tip angle	0.0022 %	0.019 %
	Wire tension	0.38 %	0.46 %
Three segments (100 → 400)	Tip angle	0.0051 %	0.041 %
	Wire tension	0.33 %	1.8 %

In short. Against two exact solutions, the solver's error is small and systematic. In large deflection it is a well-behaved discretization error that halves each time the chain of bodies is refined twofold; in small deflection the solver is closer to the exact mechanics than the linear formulas it is checked against.

3.5 It agrees with a Cosserat-rod reference model

Reference model. The reference is the Cosserat-rod model of tendon-driven continuum robots by Rao et al. [3], built on the formulation of Rucker and Webster [4]. It treats the backbone as a continuous rod and solves the force and moment balance along it, with no assumption about the final shape. It was the most accurate model in that benchmark study. The authors' open-source code was extended to catheters of several segments with different stiffnesses.

Running the reference at full resolution. The reference was run so that it solves exactly the problem posed: the pull wire passes through 100 guides per segment, so its path follows the backbone to within 0.1°; the backbone and the wire are inextensible; and the wire tension is raised gradually from zero until the commanded wire displacement is reached, so the reference follows the same equilibrium as a pull applied gradually. The solver was run on the same inextensible problem. The app itself also models the stretch of the shaft and the wire.

Catheters. Two datasets of 474 catheters each were drawn at random: one-segment catheters, and three-segment catheters with a different stiffness in each segment (Table 5). Stiffness was sampled log-uniformly and the other parameters from continuous ranges. Cases with unrealistic force or deflection, or with elastic buckling, were removed when the datasets were drawn. Each catheter was bent by one pull wire, with no friction, gravity or external load.

Exclusions. Two rules, decided by the reference alone and fixed before the solver results were compared, remove cases from the 500 one-segment and 500 three-segment catheters drawn: the reference did not reach the commanded wire displacement (2 one-segment and 6 three-segment cases), or the reference catheter curls by more than 180° along its length (24 and 19), where the tip-angle measure, which runs from 0 to 180°, no longer describes the bend. That leaves 474 one-segment and 475 three-segment catheters.

Table 5. Parameter ranges of the two datasets, taken from the data.

Parameter	One segment	Three segments
Total length	24 to 100 mm	63 to 200 mm
Bending stiffness *EI* (per segment)	6.1×10^{-5} to 4.9×10^{-3} N m^2	3.5×10^{-5} to 5.0×10^{-3} N m^2
Pull-wire offset	1.0 to 10.0 mm	0.7 to 9.9 mm
Pull-wire displacement	0.7 to 19.9 mm	0.6 to 19.7 mm
Reference tip angle	up to 179 °	up to 179 °
Reference wire tension	up to 107 N	up to 43 N
Bodies in the solver model	100	100

Result. The solver follows the reference closely for both datasets (Fig. 5, Table 6). The median tip-angle error was 0.0061 ° (one segment) and 0.0035 ° (three segments), and the median wire-tension error 0.52 % and 0.47 %. 100 % and 100 % of the catheters were within 2 ° in tip angle (Fig. 4). The tip angle is set mainly by how far the wire is pulled, so it agrees most closely. The tension carries the discretization error: the solver needs slightly more tension than the reference (bias 0.53 % and 0.53 %), the same size and sign as its departure from the exact arc in Section 3.3.

Table 6. Error of the solver against the Cosserat-rod reference. The tip-angle error is read mainly in degrees, the tension error mainly as a percentage, since the tensions span several orders of magnitude.

	One segment (474)				Three segments (475)			
	Tip angle		Tension		Tip angle		Tension	
Measure	°	%	N	%	°	%	N	%
Median	0.0061	0.0080	0.025	0.52	0.0035	0.0062	0.0060	0.47
P95	0.054	0.033	0.22	0.59	0.052	0.035	0.056	1.2
Largest	0.22	0.12	0.55	0.70	0.21	0.12	0.32	2.3
Bias	0.014	0.012	0.056	0.53	0.012	0.011	0.015	0.53

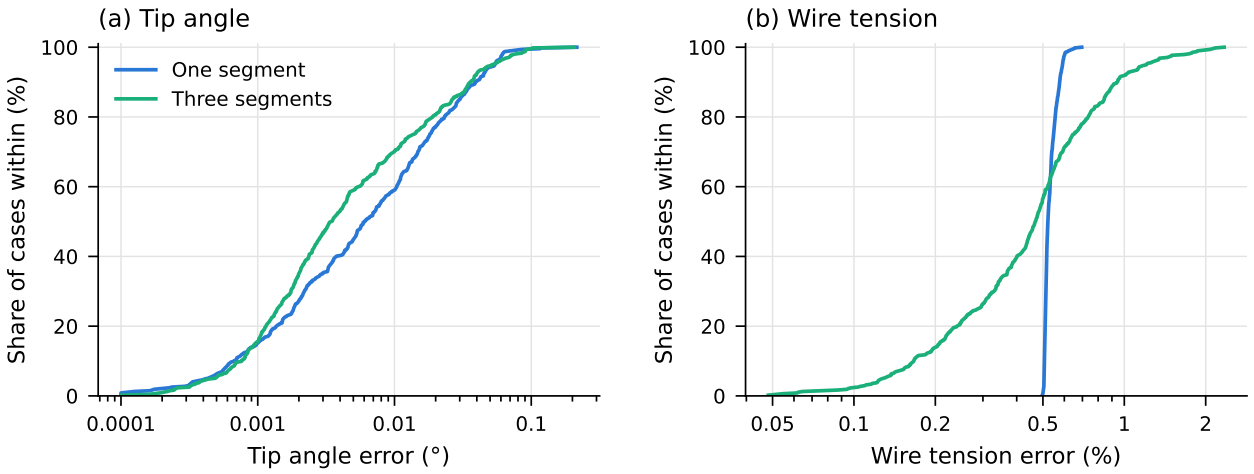

Figure 4. Share of catheters within a given error: (a) tip angle in degrees, (b) wire tension in per cent.

The largest differences. No catheter differs from the reference by more than 0.22 ° (one segment) or 0.21 ° (three segments) in tip angle, or by more than 0.70 % and 2.3 % in tension. The tension differences change little with the size of the bend; the tip-angle differences grow with it (Fig. 6): the median tip-angle difference is 0.0053 ° and 0.0031 ° below 160° and 0.055 ° and 0.061 ° beyond it. Tight bends put fewer bodies across each part of the curve, so this is where the discretization error is largest. Both models also keep to the wire kinematics: a straight wire at offset d shortens by about $d\,\theta$, and below 160° the tip angles follow that relation to a median of 0.0035 ° (reference) and 0.0017 ° (solver) for one-segment catheters.

In short. Against a high-fidelity reference, over 949 catheters in large deflection, the solver reproduces the tip angle to a median of 0.0061 ° and 0.0035 ° and the wire tension to 0.52 % and 0.47 %. No catheter differs by more than 0.22 ° in tip angle or 2.3 % in tension; the largest tip-angle differences are on the tightest bends, where the discretization error is largest.

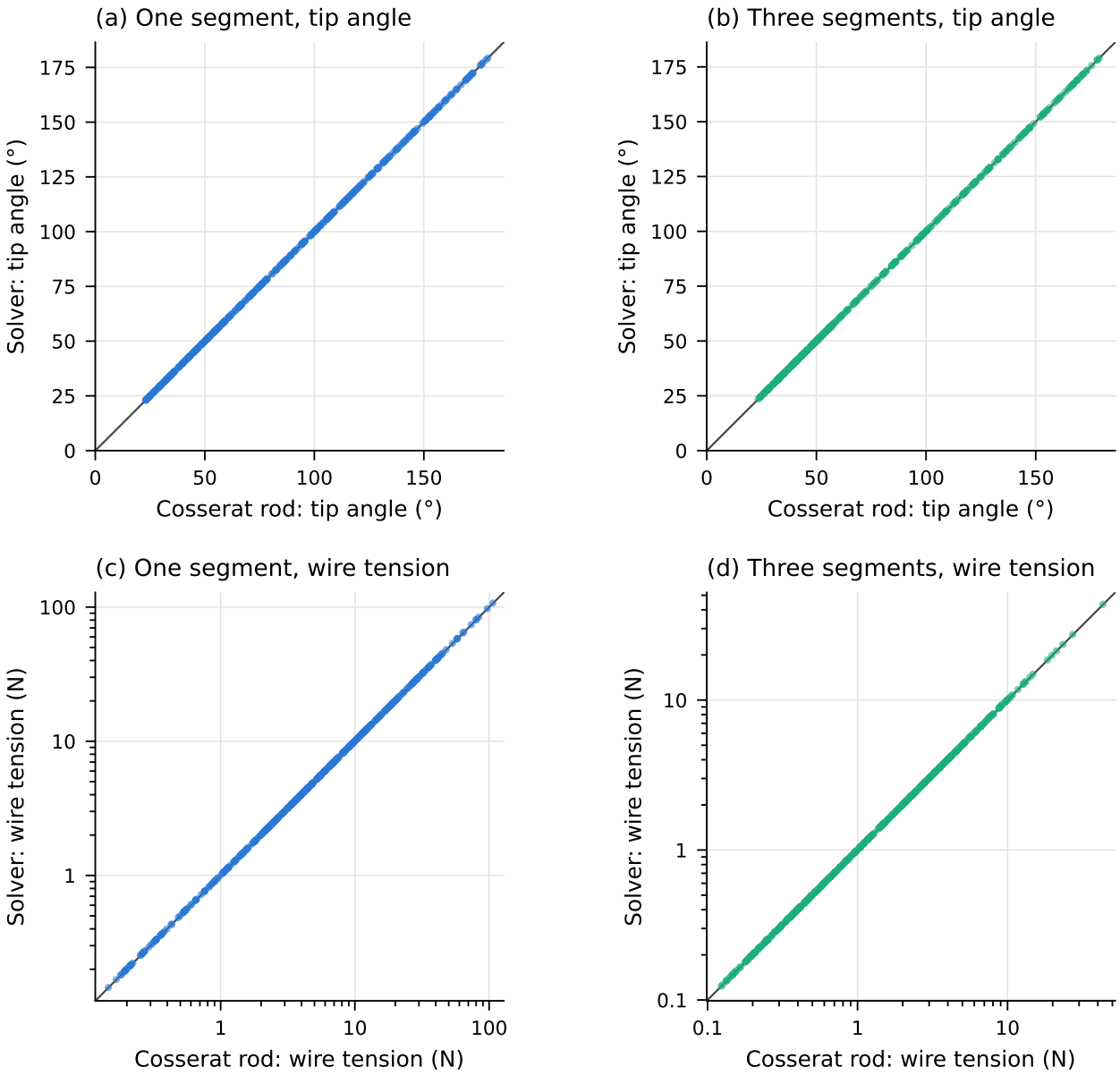

Figure 5. Solver against the Cosserat-rod reference: tip angle (a, b) and wire tension (c, d), for one-segment (left) and three-segment catheters (right). The line is equality; tensions are on logarithmic axes.

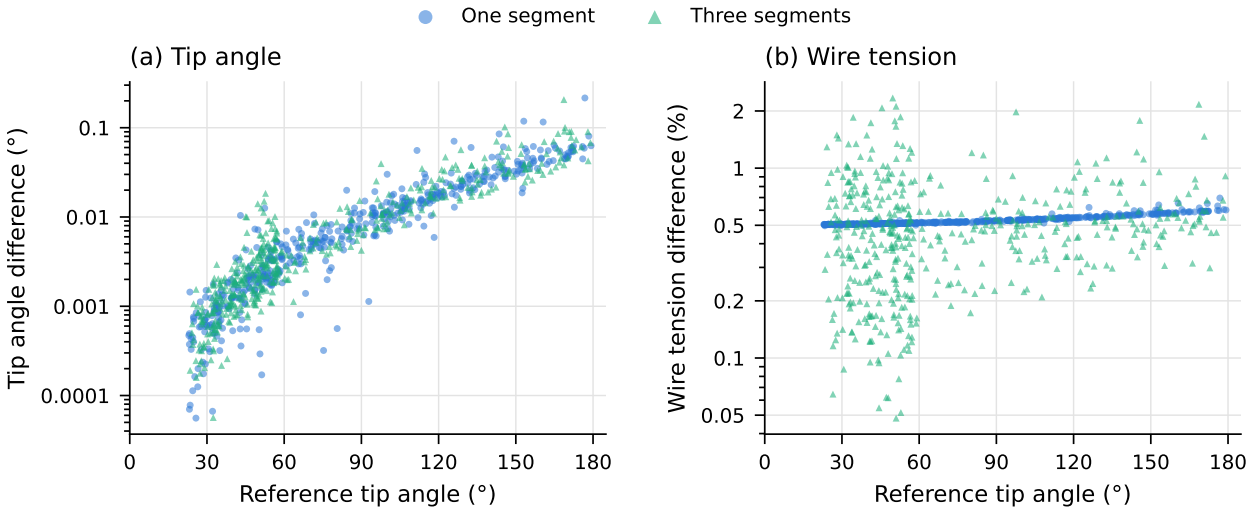

Figure 6. Difference between the solver and the reference against the reference tip angle: (a) tip angle, (b) wire tension.

4 What this means for design work

Tip angle and shape. When the pull is the input, as in the app, expect the solver's tip angle to be within a fraction of a degree of the exact mechanics of the model it solves (median 0.0061 ° and 0.0035 °, at most 0.22 °). For a given wire tension it bends about 0.51 % less than the exact arc, slightly on the stiff side. Both are small next to the uncertainty of the section stiffnesses it is given, so the stiffness inputs, not the solver, set the accuracy of a steering prediction for a real catheter.

Wire tension. The tension follows the reference with a median difference of 0.52 % (one segment) and 0.47 % (three segments). The larger differences are on the largest bends: for one-segment catheters the P95 is 0.58 % below 160° and 0.65 % beyond it.

Comparing designs. The solver's discretization error has the same sign in 100 % of catheters: it bends slightly less than the exact arc. An error with a consistent sign largely cancels when two similar designs are compared, so differences between designs carry less of it than either absolute value.

5 Limits and next steps

Limits.

- **No physical validation yet.** The checks here compare the solver with exact solutions and a reference model of the same idealised problem. They show the mechanics are solved correctly. They do not compare predicted shapes with physical catheters.
- **Scope of the reference comparison.** One pull wire, planar bending, uniform segments, an inextensible backbone and wire, and no friction, contact or external load. Bends past 180° were not compared. The app's options for wire friction, several wires and the proximal fixture are covered by the code checks only.
- **Discretization.** At the standard budget the solver bends about 0.51 % less than the exact arc (Section 3.3), about $1/(2N)$ for a chain of N bodies; a design steered with fewer bodies on a short bending section has a correspondingly larger error.
- **The app's bend limit.** In the app, a command that would bend the catheter beyond a set maximum or below a minimum bend radius is limited. The comparison lifts that limit so that every case is solved.

Next step: bench validation of steering. Build a small set of steerable shafts with measured section stiffness, pull each wire in steps, and record the tip angle (by imaging) and the wire tension (with a load cell), against predictions made before the test.

6 Technical notes

Error metrics. The error is solver − reference, in physical units and as a percentage of the reference. Median, 95th percentile (P95) and largest absolute errors are given, with the bias (the mean signed error).

Tip angle and tension. The tip angle is the rotation of the distal tip relative to the straight catheter. The tension is the force in the pull wire at equilibrium. The tip deflection is the displacement of the distal end across the straight axis.

Revision 5.1 (November 2026). Every study was re-run on VirtuCath 1.7.5, with two changes to the solver since version 5.0. First, the pull wire is routed through a guide at each joint between bodies. Before, it ran through a guide at the middle of each body, so the wire crossed each bent joint as a straight chord and its moment arm grew with the bend, by about a quarter of the body length times the joint's bend. That error matters most when the wire lies close to the centreline. In version 5.0 it partly offset the chain's own stiffening (Section 3.3) in the wire tension, and added to the tip-angle error. Second, the report now uses the body allocation of the app's Steering tab (since VirtuCath 1.7.2): 100 bodies per catheter, where version 5.0 used a minimum body length of 2 mm, which gave 11 to 99 bodies. Against the Cosserat-rod reference, the median tip-angle error went from 0.13 and 0.056 ° to 0.0061 and 0.0035 °, the median tension error from 1.0 and 0.33 % to 0.52 and 0.47 %, and the largest tip-angle and tension errors from 4.3 ° and 4.8 % to 0.22 ° and 2.3 %. The departure from the exact arc went from 1.2 % to 0.51 % and the small-deflection median from 0.18 % to 0.063 %. The refinement levels are now 200 and 400 bodies. The datasets, the reference solutions and the exclusion rules are unchanged, and every verdict is unchanged.

Data. The datasets, the solver results and the scripts that produced every number and figure in this report are held by VirtuCath LLC and are available to reviewers on request.

Glossary. **Body**: one rigid link of the chain that represents the catheter. **Bias**: the mean signed error. **Cosserat rod**: a model of a slender body as a continuous curve that carries forces and moments. **P95**: the value that 95 % of cases stay below. **Verification**: is the model solved correctly? **Validation**: is it the right model for the use?

References

[1] ASME. V&V 10-2019: Standard for verification and validation in computational solid mechanics. American Society of Mechanical Engineers; 2019.

[2] Young WC, Budynas RG. Roark's formulas for stress and strain. 7th ed. New York: McGraw-Hill; 2002.

[3] Rao P, Peyron Q, Lilge S, Burgner-Kahrs J. How to model tendon-driven continuum robots and benchmark modelling performance. Front Robot AI. 2021;7:630245. doi:10.3389/frobt.2020.630245.

[4] Rucker DC, Webster RJ III. Statics and dynamics of continuum robots with general tendon routing and external loading. IEEE Trans Robot. 2011;27:1033–1044.