Verification of the VirtuCath Kink Radius Simulation

The cross-section bending solver checked against exact beam and thin-shell solutions, a refinement study and the wire-level stiffness model

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Summary

VirtuCath's Kink Radius simulation bends one cross-section of a catheter shaft to tighter and tighter curvature and finds where it fails: where the bending moment peaks and the section starts to collapse, where the lumen has lost half its area, and where each layer first takes a permanent set, the strain past which it stays deformed. This report checks that the solver behind it solves its equations correctly. The section stiffnesses and material properties it uses are validated separately, in *Validation of the VirtuCath Stiffness Model*; the permanent-set strains are material limits, taken from each material's published or derived data, not solver outputs.

0.014 %
limit-point curvature against the exact thin-shell solution, thin walls

0.16 %
bending stiffness against exact beam theory, every wall

0.022 %
median change of the collapse onset when the mesh is doubled, 15 catheter sections

5×10^{-9}
solver tangent against finite differences

- **Exact solutions:** at small curvature the section's bending stiffness matches exact beam theory to within 0.16 %. For a thin tube, the moment peak (the Brazier limit point) converges to the exact thin-shell solution: within 0.014 % in curvature and 0.012 % in moment.
- **Numerical error:** on the sections of the bundled example designs (15 of the 17 with a collapse onset on both meshes), doubling the mesh changed it by a median of 0.022 %, and doubling the ramp steps changed it by at most 1.9 %. Sections with pull-wire lumens converge more slowly; the app meshes them more finely, and there the collapse onset is at most 1.7 % tight, that is, predicted at a tighter bend than on the finest mesh that resolves it (Section 3.3). Two sections of the 10 Fr four-way design reach a collapse onset only at the app's mesh, so its error there is not measured: on the proximal shaft every finer mesh stops at a fold of the liner beside a lumen first, and on the distal tip the moment of the finer meshes is still rising when the bore closes.
- **Wire-level model:** each braid and coil band's stiffness against flattening is set from VirtuCath's wire-level stiffness model, by a law derived from other designs. On seven thin braided and coiled catheters outside that law's data, the collapse onset falls inside the wire-level estimate for 6 of the 7 and errs on the cautious side (collapse at a gentler bend) by 2.4 % on the remaining one.
- **Not yet done:** a pre-registered comparison of predicted kink radii with physical catheters (Section 5).

In short. The Kink Radius solver solves its mechanics correctly: it reproduces exact beam and thin-shell solutions and converges under refinement. Its collapse onset is its best-verified output; on sections with pull-wire lumens it can be up to 1.7 % tight at the app's mesh, and on two sections of the 10 Fr four-way design it is not converged. Its permanent-set radii are as reliable as the permanent-set strains of the materials.

1 The Kink Radius solver

The simulation takes one cross-section of the shaft, with every layer, braid or coil band and lumen, and bends it to a uniform curvature in steps, starting from a gentle bend and tightening until it fails. A long shaft in uniform bending has the same cross-section all along its length, so a two-dimensional model of the section is exact for that loading: the section can flatten (ovalize), its walls can bend and stretch, and the axial stress carries the bending moment. Braid and coil layers are represented as equivalent bands with the stiffness of the wires in the polymer; each band's stiffness around the shaft, which resists flattening, is set so that the wall's ring stiffness matches VirtuCath's wire-level model (Section 3.4). At each step the solver finds the equilibrium shape of the section, with large deformation and large rotation, and reports:

- **Collapse onset:** the curvature at which the bending moment peaks (the Brazier limit point). Past it the section flattens faster than it gains stiffness and collapses.
- **Lumen loss:** the curvature at which the lumen has lost half its area.
- **Permanent set:** for each layer, the curvature at which it first reaches its permanent-set strain, past which it stays deformed: a lasting set, not the small residual most polymers show at gentler bends and recover from.

Each event is reported as the centerline bend radius at which it occurs. A result that places an event at a gentler bend (a larger radius) than a reference is called *cautious*; one that places it at a tighter bend (a smaller radius) is called *tight*.

Intended use. Like the rest of VirtuCath, the Kink Radius simulation supports early and mid design: screening and comparing designs. VirtuCath does not replace design verification tests.

2 How the solver was checked

The checks follow ASME V&V 10 [1] and answer five questions (Table 1).

Table 1. The evidence at a glance.

Question	How it was checked	Result	Status
1. Does the code solve its equations correctly?	Solver tangent against finite differences; automated behavior tests	All within tolerance	**Pass**
2. Does it reproduce exact solutions?	Bending stiffness against beam theory; thin-tube limit point and flattening against the exact thin-shell solution	Within 0.16 % and 0.014 %	**Pass**
3. Is the numerical error small?	Mesh, wall elements and ramp varied on uniform tubes and on 17 catheter sections	Median 0.022 %; up to 1.7 % tight on sections with pull-wire lumens; two 10 Fr sections not converged	**Pass**
4. Does it agree with the wire-level model?	Collapse onset of thin braided and coiled walls outside the data of the band hoop law	Inside the range, or up to 2.4 % cautious	**Pass**
5. Does it agree with physical catheters?	Pre-registered mandrel kink tests	–	Future work

3 Results

3.1 The code solves its equations correctly

The solver finds each step's equilibrium with Newton's method on the analytic tangent stiffness of the section. On a two-layer wall (a hyperelastic polymer layer and an orthotropic band) flattened by 15 % at a tight bend, the tangent agrees with finite differences of the out-of-balance force to 5×10^{-9} of its largest entry (criterion 10^{-6}). Automated tests also check that the collapse onset changes by less than 1 % between ramps of 15, 30 and 60 steps; that a mirror-symmetric section solved on one half gives the full-section result to a relative 10^{-6}; and that a mesh with four times as many elements changes the moment by less than 1 % and the bending stiffness by less than 0.5 %. All pass.

3.2 It reproduces exact solutions

Bending stiffness. At small curvature, a tube's bending stiffness is exactly *EI*, for a thin or a thick wall. Over every tube in this report, with a ratio r/t of mean radius to wall thickness from 5 to 100, the solver's bending stiffness at the first, gentlest step matched it to within 0.16 %. The largest difference is on the coarsest ramp, whose first step is already curved enough for the section to begin to flatten; it shrinks as the first step is made gentler.

The thin-tube limit point. A thin tube bent further flattens, and its moment peaks at the Brazier limit point. Brazier's classical formula [2] assumes a small flattening of a single shape; the thin-shell problem without that assumption was solved by Reissner and Weinitschke [3]. For this report it was solved independently, by minimizing the energy of the flattening ring with no assumption about its shape (Section 6): the moment peaks at 1.0102 times Brazier's curvature and 0.9710 times his moment, with the section flattened by 0.257. In Brazier's units the answer is the same for every thin wall, so a thin-walled tube must converge to it.

Result. The solver converges to the exact thin-shell solution as the wall gets thinner and the mesh finer (Fig. 1, Table 2): for r/t of 50 and 100 its limit point is within 0.014 % in curvature and 0.012 % in moment. Thicker walls depart from it smoothly, by an amount that grows roughly with the square of the wall thickness, as expected of a theory that leaves the thickness out; at $r/t = 5$ the peak is 0.65 % lower in curvature and 0.75 % lower in moment. At $r/t = 100$ the whole moment curve, through the peak and beyond, follows the thin-shell solution to within a few hundredths of a percent (Fig. 2). The flattening agrees too, once both are measured on the same surface: the solver reports the flattening of the outer diameter and the thin-shell solution that of the mid-surface, and this alone accounts for almost all of the lower flattening of the thicker walls in Table 2.

Table 2. The limit point of a uniform tube at the finest mesh (288 elements around), in Brazier's units, and the size of its difference from the exact thin-shell solution (curvature 1.0102, moment 0.9710, mid-surface flattening 0.257). The solver's flattening is that of the outer diameter.

Wall, r/t	Curvature	Moment	Flattening	Curvature vs exact	Moment vs exact
100	1.0104	0.9711	0.256	0.014 %	0.012 %
50	1.0102	0.9710	0.254	0.0046 %	0.0031 %
20	1.0098	0.9705	0.251	0.045 %	0.049 %
10	1.0085	0.9690	0.245	0.17 %	0.20 %
5	1.0037	0.9637	0.236	0.65 %	0.75 %

Orthotropic walls. A braided band can be much stiffer around the hoop than along the shaft, or the reverse. On walls with $r/t = 20$ and a hoop stiffness from a quarter to four times the axial stiffness, the limit point stayed within 0.88 % of the thin-shell value (1.010 and 1.001 times Brazier's curvature, in the units of each wall). At sixteen times, with a through-thickness shear modulus that is soft next to the hoop modulus, it fell to 0.952. The solver includes the shear deformation through such a wall, which thin-shell theory leaves out; in repeat runs most of the difference disappeared when the shear modulus was raised sixteenfold or the wall was made thinner.

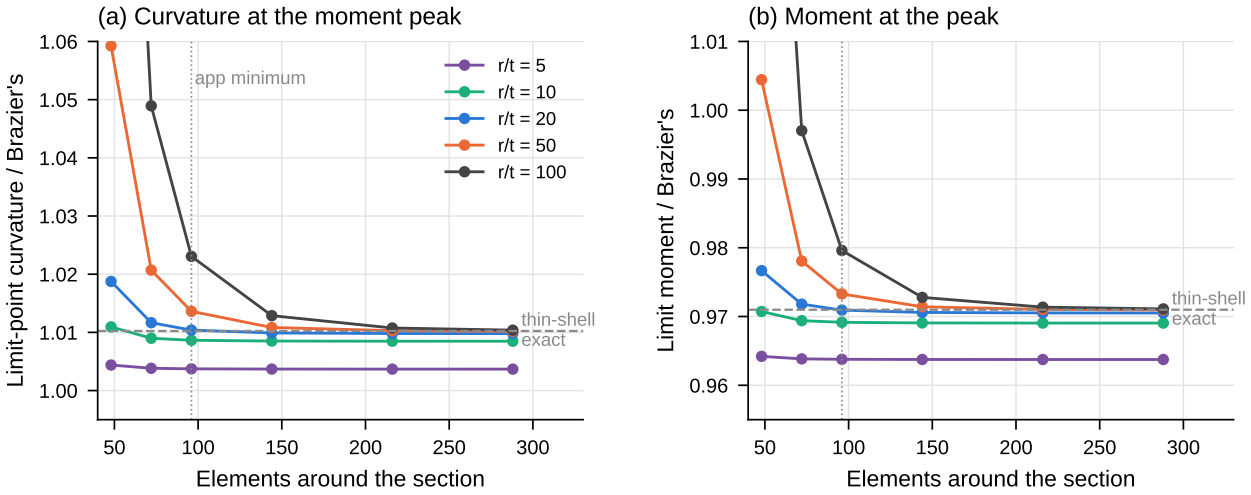

Figure 1. The limit point of a uniform tube against the number of elements around the section, for five wall thicknesses: (a) curvature and (b) moment at the peak, in Brazier's units. Dashed: the exact thin-shell solution. Dotted: the minimum mesh the app uses.

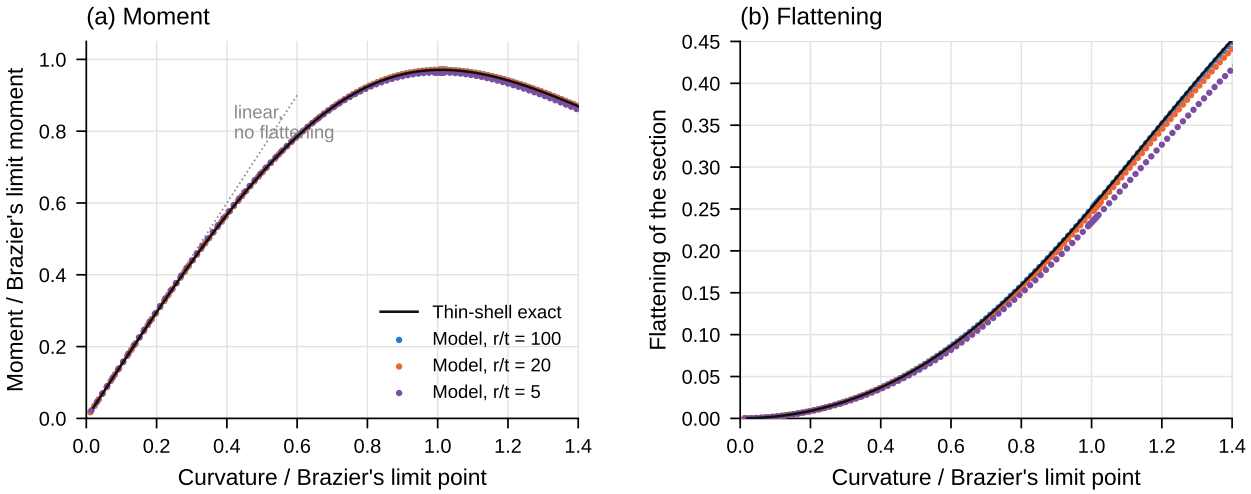

Figure 2. The whole bend of a uniform tube against the exact thin-shell solution: (a) bending moment and (b) flattening of the section, against the curvature, in Brazier's units. Dotted: linear bending with no flattening. The solver follows the exact solution through the peak; the thick wall ($r/t = 5$) peaks slightly earlier and lower. The solver's flattening is measured on the outer diameter, which places the thicker walls slightly below the mid-surface curve.

3.3 The numerical error is small

Uniform tubes. The limit point converges fast with the number of elements around the section, at an observed order of about 4.0 on the thinnest wall (Fig. 1). At the app's minimum of 96 elements it is within 0.060 % of the fully refined value for walls with r/t up to 20, which covers catheter shafts. Very thin walls need more ($r/t = 100$: 1.3 % at 96), and the app raises the count for them, to about 160 at $r/t = 100$. Refining the mesh through the two thickest walls (r/t of 5 and 10) from two to up to eight elements changed the limit point by 0.056 % or less, and changing the ramp from 20 to 160 steps ($r/t = 20$) changed it by 0.0020 %.

Catheter sections. Every section of the 6 bundled example designs except the short pull rings (17 sections) was run with the app's settings and again with 1.5 and 2 times the elements around and twice the ramp steps (Table 3).

Table 3. Change in the event radii of the example catheter sections when the run is refined, against the app's settings: median and largest change over the sections where the event occurred (number of sections in brackets).

Event	1.5× elements around	2× elements around	2× ramp steps
Collapse onset (15)	0.015 / 5.6 %	0.022 / 4.6 %	0.0049 / 1.9 %
50 % lumen loss (11)	0.020 / 1.4 %	0.020 / 1.9 %	0 / 0.0058 %
First permanent set (17)	0.012 / 1.4 %	0.027 / 2.3 %	0 / 0.0085 %

For most sections the event radii change by hundredths of a percent: for them the app's mesh and ramp are converged. The largest change in collapse onset, 4.6 %, is on a section with pull-wire lumens, the flexible section of the 10 Fr four-way design (Section 3.3). The largest change in first permanent set, 1.4 %, is on the distal tip of the large-lumen braided shaft, which first sets in its thin liner. Every example section reaches a collapse onset at the app's settings; the 10 Fr four-way's proximal shaft takes a permanent set first, in the pull-wire lumen liner at 16.6 mm. Two sections are left out of the collapse row because the finer meshes reach no onset to compare. The 10 Fr four-way's proximal shaft is described in Section 3.3. Its distal tip, which has no pull-wire lumens, collapses at 3.8 mm at the app's mesh, where its bore wall later folds through itself; at 1.5 and 2 times the elements around the walls of the bore are kept apart by their contact and the moment is still rising when the bore closes. Its 50 % lumen loss, 3.2 mm, moves by 0.44 % over the three meshes and is the result to read for that section.

Sections with pull-wire lumens. A pull-wire lumen leaves a narrow ligament of wall between itself and the central lumen, and the solution converges more slowly there. The app therefore meshes a section with lumens with at least 192 elements around instead of 96; at that mesh a section takes up to about two minutes. The region around each lumen is meshed with triangles whose edges follow every boundary between the wall's layers, so no element straddles two materials, and the walls of the central lumen are kept from passing through each other by a smooth contact. The count is a deliberate design choice: twice as many elements would bring the 9 Fr shaft's collapse onset to within 0.69 % of the finest mesh, but at two to five times the run time, so the app trades a few percent of precision on these sections for a solve time that stays practical in design work.

The two stiffest such sections, the proximal shafts of the 9 Fr bidirectional and the 10 Fr four-way designs, were followed to 1152 elements around. The 9 Fr shaft's collapse onset moved from 12.0 to 12.3 mm: at the app's mesh it is 1.7 % tight. On the 10 Fr shaft the liner around a pull-wire lumen can fold before the section collapses. A solver that steps the curvature cannot follow a fold, so this one crosses it with a short damped dynamic step and keeps only the exact equilibrium it settles into; past the fold the moment falls, and the peak before it is the collapse onset. At the app's mesh this shaft reaches its collapse onset at 5.5 mm with no fold before it and then snaps toward a closed bore. A finer mesh resolves the liner's narrow tip, where the fold forms, and folds it at a gentler bend: from 288 to 1152 elements around the run stops at that fold, at 5.6 to 5.9 mm (1.3 to 6.7 % gentler than the app's onset), before it reaches an onset, so the onset on those meshes is

not known. Its first permanent set, in the pull-wire lumen liner, was 16.6 mm on every mesh.

Braid bands beside a lumen. A smeared braid band is stiff around the tube and, through its thickness, only as stiff as its matrix. Under hoop compression such a band stops being a stable continuum once its compressive hoop stress exceeds about its through-thickness shear stiffness, and it then crimps on a wavelength set by the elements, not by the wires. Beside the pull-wire lumens of the 10 Fr four-way design this happened early in the bend, and the solve stopped with the section still round, at a radius that moved with the mesh. In the bending simulation the app therefore stiffens a braid band through its thickness to at least 1/15 of its hoop stiffness, with the hoop stiffness adjusted so that the band's ring stiffness is unchanged. Of the bundled designs this applies to the four sections with pull-wire lumens: it moved the collapse onset of the 9 Fr sections by under 1 % and let the 10 Fr flexible section reach its collapse. At 1/10 that collapse is about 8 % tighter: on such a band the last few percent of the collapse radius are a modeling choice.

Permanent set beside a lumen. The strain in a thin liner peaks where it passes between the central lumen and a pull-wire lumen, and a peak at a single point sharpens without limit as the mesh resolves the ligament. A lasting set, however, forms over a hinge of finite length. The solver therefore compares the permanent-set strain with the strain averaged along the layer over a length equal to the layer's thickness. With that measure the permanent-set radius no longer drifts with the mesh: on the 9 Fr flexible section it was 21.2 mm at the app's mesh and 21.2 mm at 1152 elements around, 0.033 % on the tight side at the app's mesh.

At the app's mesh, read the collapse onset of a section with pull-wire lumens as up to 1.7 % tight where a finer mesh reaches one. Where the liner beside a lumen folds first on a finer mesh, as on the 10 Fr proximal shaft, the section may collapse up to 6.7 % more gently than the app reports.

> **In short.** Against exact solutions the solver's error is a well-behaved discretization error that falls quickly under refinement. At the app's settings it is small next to the uncertainty of the materials and stiffnesses a section is given; the largest, on sections with pull-wire lumens, is a collapse onset up to 1.7 % tight, and on two 10 Fr sections the finer meshes reach no onset to compare.

3.4 It agrees with the wire-level model

For a thin wall, the collapse onset is set by two stiffnesses of the wall at small strain: its bending stiffness along the shaft and its ring stiffness against flattening. VirtuCath's wire-level model, which places every braid and coil wire on its real path through the polymer and underlies the stiffness model, computes both. Through the exact thin-shell solution they give a collapse onset, as a range between frictionless and welded wire crossings (Table 4).

The solver's equivalent bands take their stiffness against flattening from the same wire-level model: a law derived from the wire-level ring stiffness of many other designs gives each band the hoop stiffness that makes the wall's ring stiffness match. None of the seven catheters below is among those designs. The comparison is therefore not an independent check of the ring stiffness; it checks the solver, together with that law on designs outside its data, against the wire-level estimate.

Table 4. Collapse onset, centerline bend radius in mm (a larger radius means collapse at a gentler bend): the Kink Radius solver; the thin-shell solution with the solver's own wall stiffnesses; and the thin-shell solution with the wire-level model's stiffnesses, frictionless to welded crossings. Shafts carry their labels from *Validation of the VirtuCath Stiffness Model*: P published, T third-party, H in-house.

Catheter	Solver	Thin shell, solver's wall	Thin shell, wire-level wall
H7	35.2	34.0	30.1 – 34.3
H6	62.7	61.4	54.3 – 67.2
H1	18.4	18.1	18.0 – 19.6
P1, regular braid	11.7	10.9	9.6 – 12.3
P2, half-load braid	11.9	11.2	9.7 – 12.3
P3, diamond braid	10.6	10.0	9.0 – 11.3
T1, coil under braid	24.6	22.9	15.7 – 24.8

Result. The solver's collapse radius is 1.8 to 7.6 % larger than the thin-shell estimate from its own wall stiffnesses. That is the direction seen on the thick and orthotropic walls of Section 3.2, where the solver, which includes the wall's thickness and through-thickness shear, collapses at a gentler bend than thin-shell theory. Against the wire-level wall, the solver's collapse onset falls inside the range for 6 of the 7 catheters and is 2.4 % on the cautious side on the remaining one. The coiled shaft is inside it, at 24.6 mm against 15.7 to 24.8 mm; its equivalent coil band's stiffness against flattening is within 0.10 % of the wire-level coil's.

4 What this means for design work

Collapse onset. The collapse onset is the best-verified output: it converges to the exact thin-shell solution, is converged at the app's settings on most sections, and falls inside the wire-level estimate or errs cautious, by up to 2.4 %, on braided and coiled shafts outside the data of the band hoop law. On sections with pull-wire lumens, read it as up to 1.7 % tight at the app's mesh, or up to 6.7 % where a liner beside a lumen folds first (Section 3.3). Where the finer meshes find no onset, as on the 10 Fr tip, read the 50 % lumen loss.

Permanent set. The permanent-set radius is where a layer reaches its permanent-set strain: the strain past which it stays deformed, a lasting set rather than the small residual most polymers recover from with time. The solver computes the strain correctly; the permanent-set strain itself is a property of the material, taken from its published or derived data (elastomers keep a stiffness-based estimate until they are measured) rather than solved for, so the permanent-set radius is as reliable as that property. The strain is averaged along the layer over one layer thickness, the length of the hinge a lasting set forms over, so a set in a thin liner beside a pull-wire lumen does not drift with the mesh; at the app's mesh it is within 0.033 % (Section 3.3).

Comparing designs. On most sections the numerical error at the app's settings is a few hundredths of a percent. On the sections with pull-wire lumens, the app's mesh gave a collapse onset on the tight side of the finer mesh, so the error is of one sign. Comparisons between designs of similar construction therefore carry less of it than either absolute value; a comparison between a section with pull-wire lumens and one without can carry all of it.

5 Limits and next steps

Limits.

- **No physical validation yet.** The checks here compare the solver with exact solutions and with the wire-level model. They do not compare predicted kink radii with physical catheters.
- **Pull-wire lumens.** On sections with pull-wire lumens, the collapse onset at the app's mesh can be up to 1.7 % tighter than on the finest mesh that resolves it, on the 9 Fr bidirectional's proximal shaft. On the proximal shaft of the 10 Fr four-way design a finer mesh resolves a fold of the liner beside a lumen at a gentler bend than the app's onset, by up to 6.7 %, and stops there, so its onset on the finer meshes is not known. The first permanent set of the 9 Fr flexible section, in the liner beside the lumens, was 0.033 % tight. The app's mesh is a deliberate choice that keeps a run to about two minutes (Section 3.3). The lumens are empty: the pull wire that holds a real lumen open is not modeled.
- **Collapse not reached.** On a wall that is very stiff against flattening the bending moment can still be rising where the solver stops (none of the example sections at the app's settings). The app then reports the collapse onset as not reached, with the radius at which the solver stopped as a bound; that radius depends on the mesh. Such walls take a permanent set first, and the permanent-set radius is the result to use. Once the moment curve has flattened to 60 % of its initial slope the app also estimates the collapse from it, fitting Brazier's form to the last solved points; on the example sections that reach a collapse onset, cut at the same flattening, that estimate is within 5 % (median) and 16 % (worst) of the computed onset curvature. It assumes a smooth, Brazier-type collapse, so it can read low on a section that folds locally beside its pull-wire lumens.
- **Elastic material.** The wall is elastic: a permanent-set event marks where a layer reaches its permanent-set strain, but the layer is not modeled as yielding afterwards. Thick walls, which take a set before they flatten, are where this matters most.
- **Thick walls.** No exact solution exists for the collapse of a thick multi-layer wall; for walls with r/t below about 10, which includes many catheter sections, the solver is verified by convergence and by its agreement with beam theory, not against an exact limit point.
- **Uniform bend.** The section is bent to a uniform curvature, with no localization of the kink along the shaft. The walls of the central lumen have contact; a pull-wire lumen has none.
- **Wire-level comparison.** The comparison covers the collapse onset of thin walls only; it uses the thin-shell solution to turn the wire-level stiffnesses into an onset. The bands' ring stiffness is set from the same model, on other designs, so the comparison is not an independent check of it.

Next step: bench validation of kink radius. Bend a set of shafts and plain tubes of known construction on mandrels of decreasing size, record the first visible kink and the lumen loss, and compare them with predictions made and recorded before the test.

6 Technical notes

Brazier's units. Curvatures and moments of uniform tubes are given as multiples of Brazier's limit point, $\kappa_B = \sqrt{8/3}\,\sqrt{D/A}/r^2$ and $M_B = (2\sqrt{2}\pi/9)\,r\sqrt{12AD}$, with r the mean radius, A the wall's axial membrane stiffness and D its ring bending stiffness per unit length; for an isotropic wall these reduce to Brazier's formulas [2]. Flattening is the loss of the section's height as a fraction of its diameter, measured on the outer surface by the solver and on the mid-surface by the thin-shell solution.

Thin-shell solution. The cross-section is treated as an inextensible ring. Its energy is the axial membrane energy of the bent tube plus the bending energy of the ring. At each curvature the ring shape that minimizes this energy is found numerically, with no assumed shape, and the moment follows from the energy. The ring was refined until the limit point no longer changed in the fourth digit.

Event radii. Radii are centerline bend radii. The collapse onset is located between ramp steps by a parabola through the three steps around the moment peak.

Revision 1.4 (November 2026). The material library was revised against the manufacturers' datasheets (VirtuCath 1.7.2). On the benches of Section 3.4 the change reaches the PTFE liners (Sub-Lite-Wall modulus 400 to 727 MPa, StreamLiner OTW 474.4 to 375.8 MPa) and the nylon 12 and Vestamid Poisson's ratios. The wire-level table, the bench collapse radii and every example-section run, including the fine-mesh references of Section 3.3, were re-run on the revised library (and on the 9 Fr bidirectional example's revised braid wire); the coiled shaft is now inside its wire-level range. The stiffer Sub-Lite-Wall liner changes how the 10 Fr four-way's proximal shaft and distal tip collapse: the proximal shaft reaches its onset at the app's mesh without the fold that version 1.3 crossed, and finer meshes of both reach no onset (Table 3 and Section 3.3). In the same release, the creasing event became each layer's permanent set: its lasting set, from its own material limit (yield strain times a factor for a stress-maximum yield, yield over modulus for an offset yield and for the fluoropolymers), in place of a single crease strain per material. The text and the radii in this revision are permanent-set results. VirtuCath 1.7.3 then took the nylons' datasheet yield points and in-house shaft H7's measured nylon 12 jacket; every run in this revision is on that library. Of the results here it moves only H7: collapse onset 33.8 to 35.2 mm, wire-level range 27.9–33.2 to 30.1–34.3 mm, now 2.4 % cautious. The bundled examples have no nylon layer and are unchanged. Nothing was fitted.

Data. The solver runs, the reference solutions and the scripts that produced every number and figure in this report are held by VirtuCath LLC and are available to reviewers on request.

Glossary. **Brazier limit point**: the curvature at which a bent tube's moment peaks because it flattens. **Cautious, tight**: an event placed at a gentler (cautious) or tighter (tight) bend than a reference. **Permanent-set strain**: the strain past which a layer stays deformed (a lasting set). **Elements around**: the number of elements the mesh places around the circumference of the section. **Equivalent band**: a uniform layer with the stiffness of a braid or coil and its polymer, its hoop stiffness set from the wire-level model. **Ovalization**: flattening of the section. r/t: mean radius over wall thickness. **Snap-through**: a sudden jump of the section to a flatter shape. **Verification**: is the model solved correctly? **Validation**: is it the right model for the use?

References

[1] ASME. V&V 10-2019: Standard for verification and validation in computational solid mechanics. American Society of Mechanical Engineers; 2019.
[2] Brazier LG. On the flexure of thin cylindrical shells and other “thin” sections. Proc R Soc Lond A. 1927;116:104–114.
[3] Reissner E, Weinitschke HJ. Finite pure bending of circular cylindrical tubes. Q Appl Math. 1963;20:305–319.